

Control Design for Preventing Dengue Infections using Input-Output Linearization Method

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Control Design for Preventing Dengue Infections using Input-Output Linearization Method

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Abstract. This study deals with a theoretical study of control design to prevent transmission of dengue fever viruses. The input-output linearization method is a feedback control method that can be applied to design the control functions so that the transmission of dengue viruses can be disrupted. Fogging is considered as an eradication effort and vaccination is considered as a prevention effort. Both treatments are considered as the input controls of the method while infected human variable is considered as the output control. By treating eradication effort and prevention effort as the control inputs we design controls that can stabilize the system and lead the infected population going to zero. Some numerical schemes are presented to confirm the theoretical results and the analytical results.

1. Introduction

Infectious diseases such as dengue fever are mainly found in tropical and subtropical regions such as Indonesia. From 1968 to 2009, the World Health Organization (WHO) records that Indonesia is a country with the highest cases of dengue fever in Southeast Asia where the first case was found in Surabaya at 1968 [1]. Since the disease can cause organ damage even death, early treatment should be arranged to reduce the mortality rate caused by this disease.

Mathematical modelling is one of the powerful tools to study dengue fever problem especially to analyse different strategies that might be useful in controlling the disease. This theoretical approach can help us to understand the transmission dynamics of the viruses that can involve human and mosquito population. Various mathematical modelling and mathematical theory have been formulated and developed to study the transmission of dengue viruses in human and mosquito populations. For instance Nuraini et al., 2009 [2] and Chávez et al., 2017 [3] that examined mathematical model of dengue to identify main factors that influence the spread of the disease; Diekmann et al. [4] and Driessche & Watmough [5] that developed mathematical tools called the next generation matrix and basic reproduction number to quantify the multiplication factor of the primary infectious cases to be the secondary infectious cases. Some researchers also combined mathematical model with optimal control theory to study effects of controls such as vaccination, space spraying, and treatment cycle in reducing infectious both in human and mosquito populations [6-10]. In this research we also study the transmission of dengue viruses by combining mathematical model of the disease with the input-output linearization method to observe the dynamics of the system before and after control. This study will give another point of view regarding control study of the transmission of dengue viruses such that strategies can be identified to overcome the outbreak of disease.

This paper is organized as follow. In Section 1, we present an introduction about dengue fever and some mathematical models about this disease. In Section 2 we present derivation of linear control using input-output linearization method and in the last section we present some numerical simulations and discussion about the simulation results.

2. Control Design Using Input-Output Linearization Method

Before deriving the input control, firstly let us consider the formulation of the mathematical model of dengue disease. Formulating the mathematical model of the spread of dengue infections involves the interactions of two population, i.e. human population and mosquitos' population. The two populations have an important role in the spread of dengue viruses. In the previous study, Ningsih (2017) [10] has formulated the mathematical model for dengue by considering an integrated vector control for reducing the transmission of dengue. It assumed that the integrated vector control for reducing the transmission viruses is a combination of vaccination for susceptible human and fogging for mosquito population. By using the similar model, we have the following dimensionless system [10],

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \mu_h - \left(B\beta_{mh} \frac{N_m}{N_h} x_6 (1 - u_1) + \mu_h \right) x_1 + \theta x_3 \\ B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 (1 - u_1) - (\eta_h + \mu_h) x_2 \\ \eta_h x_2 - (\theta + \mu_h) x_3 \\ \varphi \left(1 - \frac{N_m}{k N_h} x_4 \right) (x_5 + x_6) - (\eta_A + \mu_A) x_4 \\ \eta_A x_4 - (B\beta_{hm} x_2 + \mu_m + u_2) x_5 \\ B\beta_{hm} x_2 x_5 - (\mu_m + u_2) x_6 \end{bmatrix}, \quad (1)$$

Table 1. Variables and parameters of the model (1).

Parameter	Description	Value	Ref.
$1/\mu_h$	average lifespan of humans (days)	71 * 365	[6]
N_h	total of human population	48000	Assumed
N_m	total of mosquito population	500000	Assumed
B	average number of bites on humans by mosquitoes, per day	0.8	[6]
β_{mh}	transmission probability from infected human (per bite)	0.35	[6]
β_{hm}	transmission probability from infected mosquito (per bite)	0.375	[6]
θ	decreasing rate of human immunity per day	0.05	Assumed
$1/\eta_h$	mean of viremia period (in days)	3	[6]
η_A	maturation rate from larvae to adult (per day)	0.08	[6]
φ	number of eggs at each deposit per capita (per day)	10	Assumed
k	number of dismissed larvae per human	6	Assumed
$1/\mu_A$	natural mortality of larvae (in day)	4	[6]
$1/\mu_m$	average lifespan of adult mosquitoes (in days)	10	[6]

where $\mathbf{x}_1(t)$ represents the proportion of susceptible humans at time t, $\mathbf{x}_2(t)$ is the proportion of infected humans with dengue virus at time t, $\mathbf{x}_3(t)$ is the proportion of humans recovered from dengue hemorrhagic fever at time t, $\mathbf{x}_4(t)$ is the proportion of mosquitoes in the water phase including eggs,

larvae, and pupae, $x_5(t)$ is the proportion of susceptible mosquitoes that do not carry dengue virus, and $x_6(t)$ is the proportion of dengue virus-infected mosquitoes that can spread the virus to human. Definitions of all variables and parameters of the model (1) are given in Table 1.

Next, we will derive the input control for vaccination and fogging by defining the infected population as the output control, $y = x_2$.

2.1. Control design for effect of vaccination strategy (u_1)

For designing the vaccination control, we assume that $u_2 = 0$. By using that assumption we can rewrite system (1) becomes

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u_1,$$

with

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mu_h - B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - \mu_h x_1 + \theta x_3 \\ B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \\ \eta_h x_2 - (\theta + \mu_h) x_3 \\ \varphi \left(1 - \frac{N_m}{kN_h} x_4 \right) (x_5 + x_6) - (\eta_A + \mu_A) x_4 \\ \eta_A x_4 - (B\beta_{hm} x_2 + \mu_m) x_5 \\ B\beta_{hm} x_2 x_5 - \mu_m x_6 \end{bmatrix}, \quad \mathbf{g}(\mathbf{x}) = \begin{bmatrix} B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 \\ -B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

For $i = 1$, we have the Lie derivative (see [11-12] for detail about Lie derivative) as follows,

$$L_g L_f^{i-1} h(\mathbf{x}) = L_g L_f^{i-1} h(\mathbf{x}) = L_g h(\mathbf{x}),$$

with

$$L_g h(\mathbf{x}) = \frac{\partial h}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = \frac{\partial x_2}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = [0 \ 1 \ 0 \ 0 \ 0 \ 0] \cdot \begin{bmatrix} B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 \\ -B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ = -B\beta_{mh} \frac{N_m}{N_h} x_1 x_6.$$

Since $L_g h(\mathbf{x}) \neq 0$ then we have the relative degree for system (1) with output control $y = x_2$ is equal to 1. Therefore we have the transformation system as follow

$$T(x) = \begin{bmatrix} z_1 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix}, \tag{2}$$

with $z_1 = x_2$ and ϕ_1, \dots, ϕ_5 that fulfils

$$\frac{\partial \phi_i}{\partial \mathbf{x}} \mathbf{g}(\mathbf{x}) = 0, \quad i = 1, \dots, 5. \quad (3)$$

Suppose we chose functions $\phi_1 = x_3, \phi_2 = x_4, \phi_3 = x_5, \phi_4 = x_6$, and $\phi_5 = x_1 + x_2$. It is easy to show that the chosen functions fulfil equation (3) such that we get the transformation system,

$$T(\mathbf{x}) = \begin{bmatrix} z_1 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_2 \\ x_4 \\ x_5 \\ x_6 \\ x_1 + x_2 \end{bmatrix} \quad (4)$$

From equation (4) we get

$$\begin{aligned} \dot{z}_1 &= \dot{x}_2, \\ \dot{\phi}_1 &= \dot{x}_3, \\ \dot{\phi}_2 &= \dot{x}_4, \\ \dot{\phi}_3 &= \dot{x}_5, \\ \dot{\phi}_4 &= \dot{x}_6, \\ \dot{\phi}_5 &= \dot{x}_1 + \dot{x}_2, \end{aligned} \quad (5)$$

where v_1 is the new input, $v_1 = -c_0 z_1$ with positive parameter c_0 . If the system (1) is substituted to equation (5) then we get

$$\begin{aligned} \dot{z}_1 &= v_1 \\ \dot{\phi}_1 &= \eta_h x_2 - (\theta + \mu_h) x_3, \\ \dot{\phi}_2 &= \varphi \left(1 - \frac{N_m}{kN_h} x_4 \right) (x_5 + x_6) - (\eta_A + \mu_A) x_4, \\ \dot{\phi}_3 &= \eta_A x_4 - (B\beta_{hm} x_2 + \mu_m) x_5, \\ \dot{\phi}_4 &= B\beta_{hm} x_2 x_5 - \mu_m x_6, \\ \dot{\phi}_5 &= \mu_h - \mu_h x_1 + \theta x_3 - (\eta_h + \mu_h) x_2. \end{aligned} \quad (6)$$

If we rewrite system (6) using the transformation variables in (2) then we have the normal system of (1) with output control $y = x_2$ as follow

$$\begin{aligned} \dot{z}_1 &= v_1, \\ \dot{\phi}_1 &= \eta_h z_1 - (\theta + \mu_h) \phi_1, \\ \dot{\phi}_2 &= \varphi \left(1 - \frac{N_m}{kN_h} \phi_2 \right) (\phi_3 + \phi_4) - (\eta_A + \mu_A) \phi_2, \\ \dot{\phi}_3 &= \eta_A \phi_2 - (B\beta_{hm} z_1 + \mu_m) \phi_3, \\ \dot{\phi}_4 &= B\beta_{hm} z_1 \phi_3 - \mu_m \phi_4, \\ \dot{\phi}_5 &= \mu_h - \mu_h (\phi_5 - z_1) + \theta \phi_1 - (\eta_h + \mu_h) z_1, \\ y &= h(\mathbf{x}) = z_1. \end{aligned} \quad (7)$$

If $z_1 = 0$ then we get the zero dynamics as follow

$$\begin{aligned}
 \dot{\phi}_1 &= -(\theta + \mu_h)\phi_1, \\
 \dot{\phi}_2 &= \varphi \left(1 - \frac{N_m}{kN_h} \phi_2 \right) (\phi_3 + \phi_4) - (\eta_A + \mu_A)\phi_2, \\
 \dot{\phi}_3 &= \eta_A \phi_2 - \mu_m \phi_3, \\
 \dot{\phi}_4 &= -\mu_m \phi_4, \\
 \dot{\phi}_5 &= \mu_h - \mu_h \phi_5 + \theta \phi_1.
 \end{aligned} \tag{8}$$

By using linearization method, it is easy to show that the zero dynamic is a stable system. Furthermore, we can formulate the control function for u_1 using the following term [11],

$$u_1 = \frac{1}{L_g h(\mathbf{x})} \cdot [-L_f h(\mathbf{x}) + v_1].$$

Consider

$$\begin{aligned}
 L_g h(\mathbf{x}) &= \frac{\partial h}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = \frac{\partial x_2}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot \begin{bmatrix} B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 \\ -B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = -B\beta_{mh} \frac{N_m}{N_h} x_1 x_6, \\
 L_f h(\mathbf{x}) &= \frac{\partial h}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x}) = \frac{\partial x_2}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x}) = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot \begin{bmatrix} \mu_h - B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - \mu_h x_1 + \theta x_3 \\ B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \\ \eta_h x_2 - (\theta + \mu_h) x_3 \\ \varphi \left(1 - \frac{N_m}{kN_h} x_4 \right) (x_5 + x_6) - (\eta_A + \mu_A) x_4 \\ \eta_A x_4 - (B\beta_{hm} x_2 + \mu_m) x_5 \\ B\beta_{hm} x_2 x_5 - \mu_m x_6 \end{bmatrix} \\
 &= B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2.
 \end{aligned}$$

Therefore we get the control function for u_1 ,

$$u_1 = \frac{B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 + c_0 z_1}{B\beta_{mh} \frac{N_m}{N_h} x_1 x_6}. \tag{9}$$

2.2. Control design for effect of fogging strategy (u_2)

By assuming $u_1 = 0$ and using the similar way as before, we get the relative degree as follow. For $i=1$, we get

$$L_g L_f^{i-1} h(\mathbf{x}) = L_g h(\mathbf{x}),$$

with

$$L_g h(\mathbf{x}) = \frac{\partial h}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = \frac{\partial x_2}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = [0 \ 1 \ 0 \ 0 \ 0 \ 0] \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -x_5 \\ -x_6 \end{bmatrix} = 0.$$

Since we have $L_g h(\mathbf{x}) = 0$, it means that we have to continue our quantification. For $i = 2$

$$L_g L_f^{2-1} h(\mathbf{x}) = L_g L_f h(\mathbf{x}),$$

with

$$L_f h(\mathbf{x}) = \frac{\partial h}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x}) = \frac{\partial x_2}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x}) = [0 \ 1 \ 0 \ 0 \ 0 \ 0] \cdot \begin{bmatrix} \mu_h - B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - \mu_h x_1 + \theta x_3 \\ B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \\ \eta_h x_2 - (\theta + \mu_h) x_3 \\ \varphi \left(1 - \frac{N_m}{kN_h} x_4 \right) (x_5 + x_6) - (\eta_A + \mu_A) x_4 \\ \eta_A x_4 - (B\beta_{hm} x_2 + \mu_m) x_5 \\ B\beta_{hm} x_2 x_5 - \mu_m x_6 \end{bmatrix}$$

$$= B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2.$$

So that we get

$$L_g L_f h(\mathbf{x}) = \frac{\partial L_f h(\mathbf{x})}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = \frac{\partial \left(B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \right)}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x})$$

$$= \begin{bmatrix} B\beta_{mh} \frac{N_m}{N_h} x_6 & -(\eta_h + \mu_h) & 0 & 0 & 0 & B\beta_{mh} \frac{N_m}{N_h} x_1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -x_5 \\ -x_6 \end{bmatrix} = -B\beta_{mh} \frac{N_m}{N_h} x_1 x_6.$$

Since $L_g L_f h(\mathbf{x}) \neq 0$ then we get relative degree with output control $y = x_2$ is equal to 2. Because of the relative degree of the system is equal to 2 then we have the transformation system

$$T(x) = \begin{bmatrix} z_1 \\ z_2 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix},$$

where ϕ_1, \dots, ϕ_4 satisfies equation (3). Suppose we chose $\phi_1 = x_1, \phi_2 = x_3, \phi_3 = x_4, \phi_4 = x_1 + x_2$. It is easy to show that the chosen functions fulfil equation (3) such that we get the transformation system,

$$T(x) = \begin{bmatrix} z_1 \\ z_2 \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h)x_2 \\ x_1 \\ x_3 \\ x_4 \\ x_1 + x_2 \end{bmatrix}. \quad (10)$$

Using equation (10) we have

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= v_2, \\ \dot{\phi}_1 &= \dot{x}_1, \\ \dot{\phi}_2 &= \dot{x}_3, \\ \dot{\phi}_3 &= \dot{x}_4, \\ \dot{\phi}_4 &= \dot{x}_1 + \dot{x}_2. \end{aligned} \quad (11)$$

where $v_2 = -c_0 z_1 - c_1 z_2$ is the new input control with positive parameters c_0 and c_1 . If we substitute equation (1) into equation (11) then we get

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= v_2, \\ \dot{\phi}_1 &= \mu_h - \left(B\beta_{mh} \frac{N_m}{N_h} x_6 + \mu_h \right) x_1 + \theta x_3, \\ \dot{\phi}_2 &= \eta_h x_2 - (\theta + \mu_h) x_3, \\ \dot{\phi}_3 &= \varphi \left(1 - \frac{N_m}{kN_h} x_4 \right) (x_5 + x_6) - (\eta_A + \mu_A) x_4, \\ \dot{\phi}_4 &= \mu_h - \mu_h x_1 + \theta x_3 - (\eta_h + \mu_h) x_2. \end{aligned} \quad (12)$$

If we rewrite system (12) using the transformation variables in (10) then we have the normal system of (1) with output control $y = x_2$ as follow

$$\begin{aligned}
 \dot{z}_1 &= z_2, \\
 \dot{z}_2 &= v_2, \\
 \dot{\phi}_1 &= \mu_h - \mu_h \phi_1 + \theta \phi_2, \\
 \dot{\phi}_2 &= \eta_h z_1 - (\theta + \mu_h) \phi_2, \\
 \dot{\phi}_3 &= \varphi \left(1 - \frac{N_m}{kN_h} \phi_3 \right) (a) - (\eta_A + \mu_A) \phi_3, \\
 \dot{\phi}_4 &= \mu_h - \mu_h \phi_1 + \theta \phi_2 - \eta_h z_1 - \mu_h z_1, \\
 y &= h(x) = z_1.
 \end{aligned} \tag{13}$$

If $z_1 = z_2 = 0$ then we get the zero dynamics as follow

$$\begin{aligned}
 \dot{\phi}_1 &= \mu_h - \mu_h \phi_1 + \theta \phi_2, \\
 \dot{\phi}_2 &= -(\theta + \mu_h) \phi_2, \\
 \dot{\phi}_3 &= \varphi \left(1 - \frac{N_m}{kN_h} \phi_3 \right) (a) - (\eta_A + \mu_A) \phi_3, \\
 \dot{\phi}_4 &= \mu_h - \mu_h \phi_1 + \theta \phi_2.
 \end{aligned} \tag{14}$$

By using linearization method, it is easy to show that the zero dynamic (14) is a stable system. Therefore we can formulate the control function for u_2 using the following function [11],

$$u_2 = \frac{1}{L_g L_f h(\mathbf{x})} \cdot \left[-L_f^2 h(\mathbf{x}) + v_2 \right] \tag{15}$$

with

$$L_g L_f h(\mathbf{x}) = \frac{\partial L_f h(\mathbf{x})}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}).$$

Consider that

$$L_f h(\mathbf{x}) = \frac{\partial h}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x}) = \frac{\partial x_2}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x}) = B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2.$$

Therefore we have

$$\begin{aligned}
 L_g L_f h(\mathbf{x}) &= \frac{\partial L_f h(\mathbf{x})}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) = \frac{\partial \left(B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \right)}{\partial \mathbf{x}} \cdot \mathbf{g}(\mathbf{x}) \\
 &= \begin{bmatrix} B\beta_{mh} \frac{N_m}{N_h} x_6 & -(\eta_h + \mu_h) & 0 & 0 & 0 & B\beta_{mh} \frac{N_m}{N_h} x_1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -x_5 \\ -x_6 \end{bmatrix} \\
 &= -B\beta_{mh} \frac{N_m}{N_h} x_1 x_6
 \end{aligned}$$

and

$$L_f^2 h(\mathbf{x}) = \frac{\partial L_f h(\mathbf{x})}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x}) = \frac{\partial \left(B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \right)}{\partial \mathbf{x}} \cdot \mathbf{f}(\mathbf{x})$$

$$= \begin{bmatrix} B\beta_{mh} \frac{N_m}{N_h} x_6 & -(\eta_h + \mu_h) & 0 & 0 & 0 & B\beta_{mh} \frac{N_m}{N_h} x_1 \end{bmatrix} \cdot \begin{bmatrix} \mu_h - B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - \mu_h x_1 + \theta x_3 \\ B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \\ \eta_h x_2 - (\theta + \mu_h) x_3 \\ \varphi \left(1 - \frac{N_m}{kN_h} x_4 \right) (x_5 + x_6) - (\eta_A + \mu_A) x_4 \\ \eta_A x_4 - (B\beta_{hm} x_2 + \mu_m) x_5 \\ B\beta_{hm} x_2 x_5 - \mu_m x_6 \end{bmatrix}$$

$$= B\beta_{mh} \frac{N_m}{N_h} x_6 \left(\mu_h - B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - \mu_h x_1 + \theta x_3 \right) - (\eta_h + \mu_h) \left(B\beta_{mh} \frac{N_m}{N_h} x_1 x_6 - (\eta_h + \mu_h) x_2 \right)$$

$$+ B\beta_{mh} \frac{N_m}{N_h} x_1 (B\beta_{hm} x_2 x_5 - \mu_m x_6).$$

Suppose $L_f^2 h(\mathbf{x}) = f_2$ then we have

$$u_2 = \frac{f_2 + c_0 z_1 + c_1 z_2}{B\beta_{mh} \frac{N_m}{N_h} x_1 x_6} \tag{16}$$

3. Numerical Results and Discussion

In this section we present numerical simulation of system (1) by considering different cases to evaluate effect of the controls applied to the system. For simulation purposes we use parameter values listed in Table 1. We assume that the initial conditions of the system in dimensionless term are $x_1(0) = 0.9450$; $x_2(0) = 0.0050$; $x_3(0) = 0.005$; $x_4(0) = 0.0001$; $x_5(0) = 0.0001$; $x_6(0) = 0.001$ with $N_h = 48000$ and $N_m = 500000$. Furthermore we chose $c_1 = c_2 = 1$ and the final time observation is $t_f = 100$ days. Solution of system (1) is approximated using Runge-Kutta fourth order numerical scheme (see [13] for detail about the method). We consider three cases: $u_1 \neq 0, u_2 = 0$; $u_1 = 0, u_2 \neq 0$; and $u_1 \neq 0, u_2 \neq 0$.

Case 1: $u_1 \neq 0, u_2 = 0$

In this case, we assume that control is only applied to the human population to quantify effect of vaccination without controlling mosquito population. Since u_1 is defined as effect of vaccination to the susceptible population (in dimensionless unit), we restrict its value in the bounded region $0 \leq u_1(t) \leq 1$. By setting $u_2 = 0$ we get the simulation results for human population depicted in Figure 1 and for human mosquito population depicted in Figure 2. In Figure 1-(a) we can observe that effect of vaccination is relatively small in reducing number of infected population. For instance without control, the maximal infected population about 25% while after control the maximal number of infected population decreases only about 1% (see Figure 1-(b)). So does for the recovered population, it only decreases around 1%. Conversely, effects of vaccination can be observed in the mosquito populations. Figure 2-(a) shows that number of susceptible mosquito decreases from 32% becomes 25%. While in Figure 2-(b) we can observe that number of infected mosquito also decreases about 2% at 40 days.

Case 2: $u_1 = 0, u_2 \neq 0$

In this case we assume that control only applies for mosquito population. We assume that there is no vaccination for susceptible human. Therefore we have $u_1 = 0, 0 \leq u_2(t) \leq 1$. Numerical results for this case are given in Figure 3 and 4. Figure 3 presents the proportion of susceptible human, the proportion of infected human, and the proportion of recovered human. We can observe that control applied for mosquito population significantly affects the reduction of infected human. The maximum number of infected human significantly decreases. As time is increased the number of infected human goes to zero. Conversely, as time is increased the number of susceptible human is also increase implying that almost all susceptible humans are free from infection when mosquito population is controlled. In Figure 4 we can observe that the same interpretation is applied to the mosquito populations. When mosquito population is controlled using fogging treatment, effects of this treatment significantly decrease not only infected mosquito but also susceptible mosquito and its larvae.

Case 3: $u_1 \neq 0, u_2 \neq 0$

In case three, both populations are controlled to quantify effect of simultaneous control. Here we assume that $u_1 \neq 0, u_2 \neq 0$. Simulation results for this case are depicted in Figure 5 and 6. We can observe that the simulation results in this case are not different from the previous results in case 2. Vaccination did not give significant effect to reduce infection in both human and mosquito populations. It means that the best effective way to reduce the number of infection caused by dengue viruses is by controlling the mosquito population only. This is due to control of the human population through vaccination does not have a significant effect in reducing the number of infected humans. This is also become the conclusion of this study that can be recommended to the policy maker such that outbreak of dengue fever can be avoided.

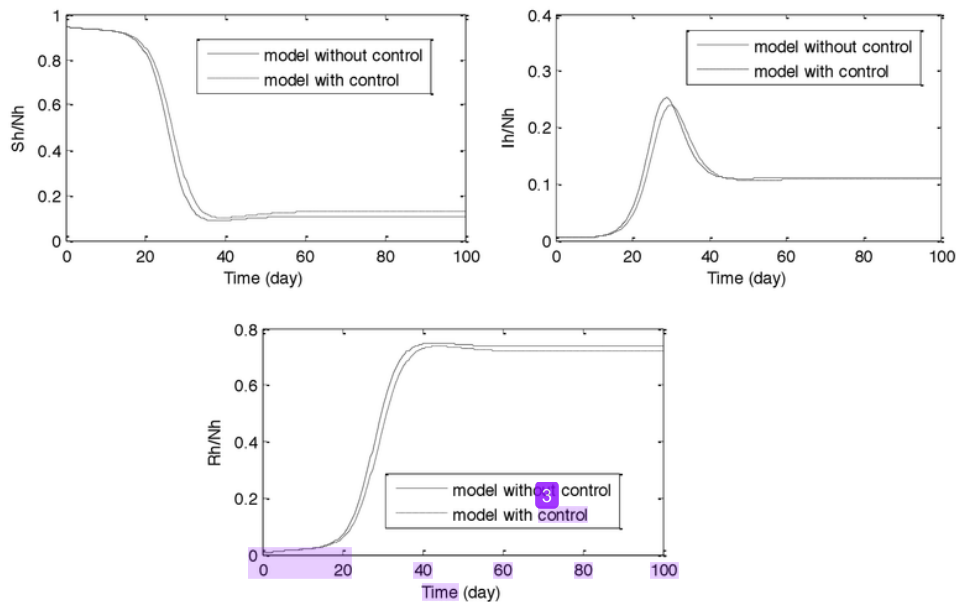


Figure 1. Numerical solutions for susceptible human (top-left), infected human (top-right) and recovered human (bottom) with and without vaccination control (case 1).

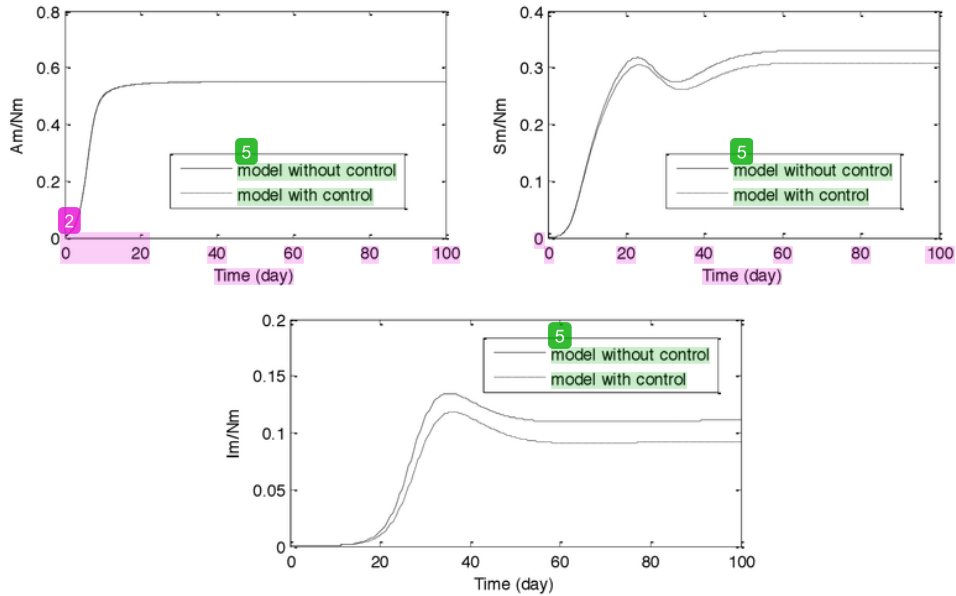


Figure 2. Numerical solutions for larvae (top-left), susceptible mosquito (top-right), and infected mosquito (bottom) with and without vaccination control (case 1).

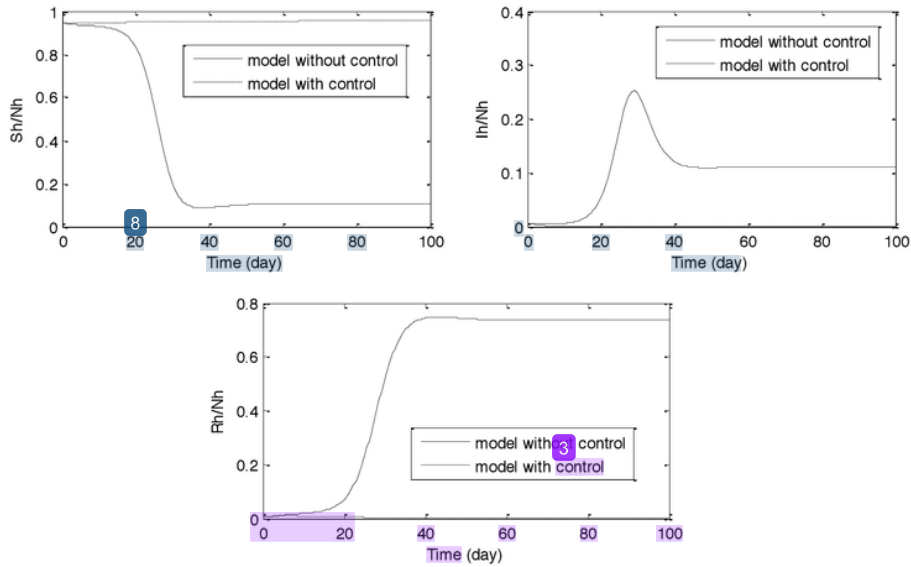


Figure 3. Numerical solutions for susceptible human (top-left), infected human (top-right) and recovered human (bottom) with and without fogging control (case 2).

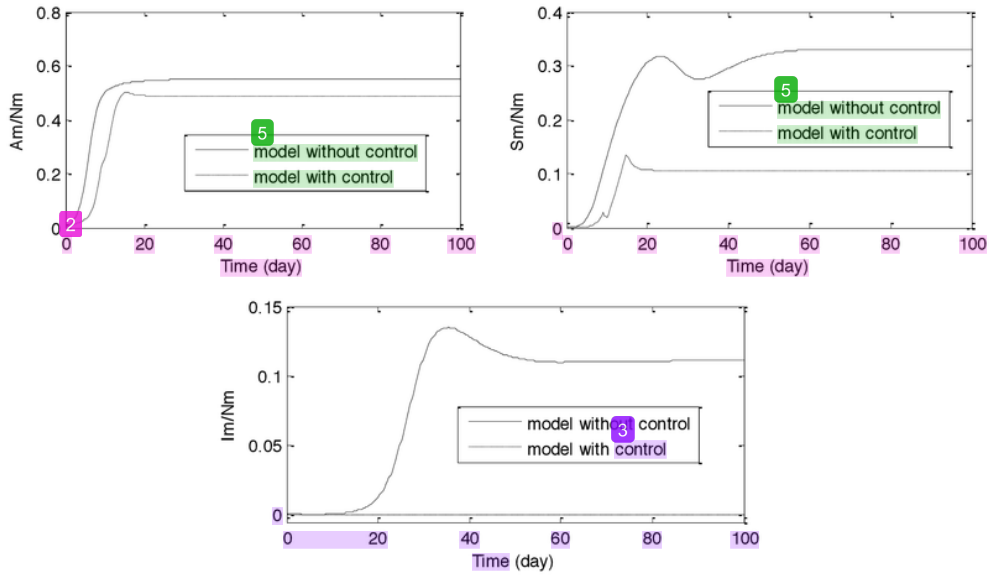


Figure 4. Numerical solutions for larvae (top-left), susceptible mosquito (top-right), and infected mosquito (bottom) with and without fogging control (case 2).

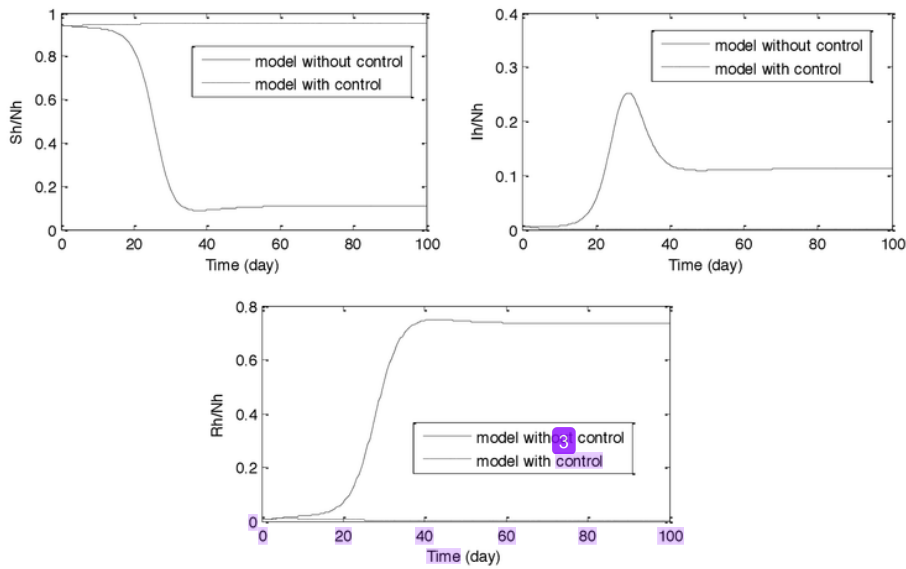


Figure 5. Numerical solutions for susceptible human (top-left), infected human (top-right) and recovered human (bottom) with and without vaccination and fogging controls (case 3).

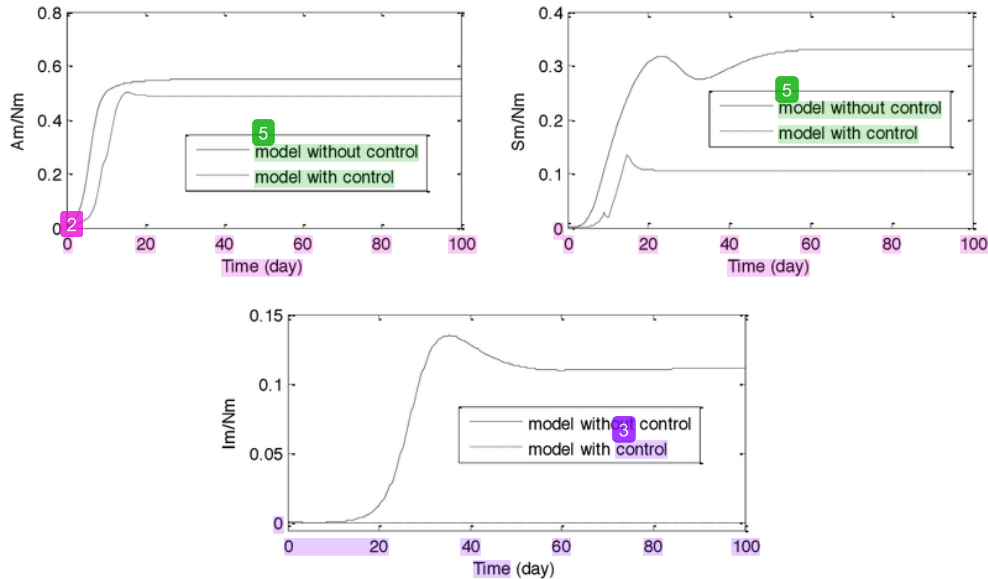


Figure 6. Numerical solutions for larvae (top-left), susceptible mosquito (top-right), and infected mosquito (bottom) with and without vaccination and fogging controls (case 3).

10 conclusions

In this paper we derived a mathematical model describing the transmission of dengue infections that involved two population, i.e. human and mosquito populations. For controlling the spread of dengue viruses, an integrated vector control was taken into consideration in the model by combining effect of vaccination for susceptible human and fogging for mosquito population. The two controls were designed by using a feedback control as the input-output linearization method. Fogging was considered as an input control with an eradication effort and vaccination was also considered as an input control with prevention effort. On the other hand, the infected human variable was considered as the output control that should be minimized. Analytically, we found the closed form of the controls in term of functions in time that depended on the model parameters. Numerically, we presented simulations for different cases and we found that vaccination did not give significant effect to reduce the infection in human population. Based on the simulation results we concluded that the best effective way to reduce the number of infection caused by dengue viruses was by controlling the mosquito population only. By controlling the mosquito population, outbreak of dengue fever can be avoided.

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